Benha University | **Electrical Circuits (2)** | **Electrical Eng. Dept.**
Faculty of Engineering | | **1st year communication**
Shoubra | | **2016/2017**

<u>Sheet (3 solutions)... Series Resonance</u>

1. A series RLC network has R=2kΩ, L=40 mH, and C=1μF. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\ \text{krad/s}$$

$$\mathbf{Z}(\omega_0) = R = \underline{\mathbf{2\ k\Omega}}$$

$$\mathbf{Z}(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$\mathbf{Z}(\omega_0/4) = \underline{\mathbf{2 - j0.75\ k\Omega}}$$

$$\mathbf{Z}(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/2) = 200 + j(100 - 2000/5)$$

$$\mathbf{Z}(\omega_0/2) = \underline{\mathbf{2 - j0.3\ k\Omega}}$$

$$\mathbf{Z}(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$\mathbf{Z}(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(2\omega_0) = \underline{\mathbf{2 + j0.3\ k\Omega}}$$

$$\mathbf{Z}(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$\mathbf{Z}(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(4\omega_0) = \underline{\mathbf{2 + j0.75\ k\Omega}}$$

Good Luck

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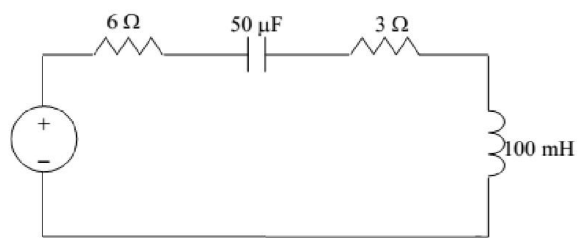
2. A coil with resistance 3Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6Ωand a signal generator that gives 110 V rms at all frequencies. Calculate ωo, Q, and B at resonance of the resultant series RLC circuit.



$$R = 6 + 3 = 9\,\Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}\,x50x10^{-12}}} = \underline{447.21\text{ krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3\,x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90\text{ rad/s}}$$

3. Design a series RLC circuit with B=20 rad/s and ω_0=1000 rad/s. Find the circuit's Q.

Let $R = 10\,\Omega$.

$$L = \frac{R}{B} = \frac{10}{20} = 0.5\text{ H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2(0.5)} = 2\,\mu\text{F}$$

$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if $R = 10\,\Omega$ then

$L = \underline{\mathbf{0.5\ H}}$, $C = \underline{\mathbf{2\ \mu F}}$, $Q = \underline{\mathbf{50}}$

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4. Consider the circuit of Figure 1
   a. Determine the values of R and C such that the circuit has a resonant frequency of 25 kHz and an rms current of 25 mA at resonance.
   b. Calculate the power dissipated by the circuit at resonance.
   c. Determine the phasor voltages, V_C, V_L, and V_R at resonance.

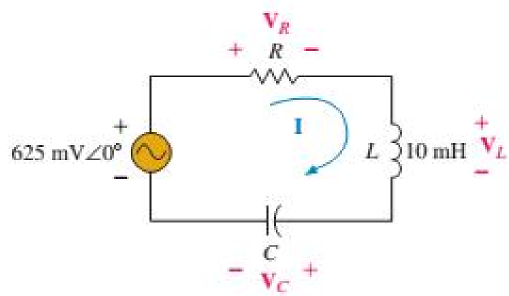
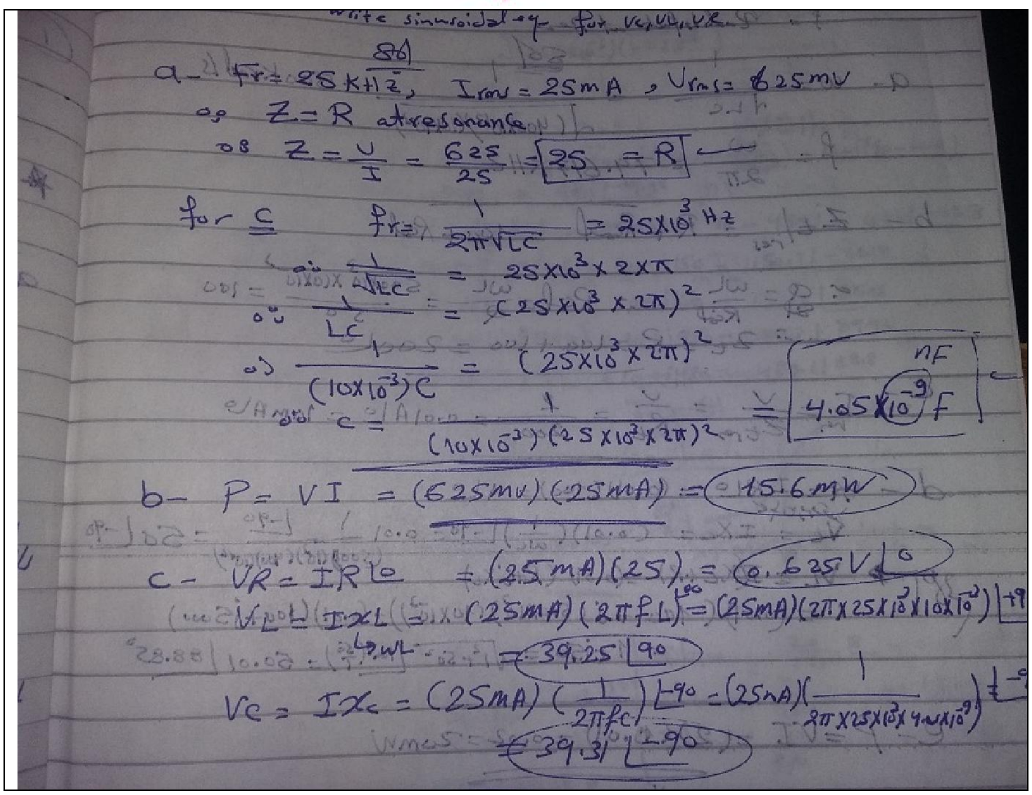


Figure (1)



write sinusoidal eq for V_C, V_L, V_R

Sol

a- $f_r = 25\ kHz$, $I_{rms} = 25\ mA$, $V_{rms} = 625\ mV$

$\therefore Z = R$ at resonance

$\therefore Z = \frac{V}{I} = \frac{625}{25} = \boxed{25 = R}$

for $\underline{C}$ $\quad f_r = \frac{1}{2\pi\sqrt{LC}} = 25\times10^{3}\ Hz$

$\therefore \frac{1}{\sqrt{LC}} = 25\times10^{3}\times2\times\pi$

$\therefore \frac{1}{LC} = (25\times10^{3}\times2\pi)^2$

$\therefore \frac{1}{(10\times10^{-3})C} = (25\times10^{3}\times2\pi)^2$

$C = \frac{1}{(10\times10^{-3})(25\times10^{3}\times2\pi)^2} = \boxed{4.05\times10^{-9}\ F}$ nF

b- $P = VI = (625mV)(25mA) = 15.6\ mW$

c- $V_R = IR\angle0 = (25mA)(25) = 0.625V\angle0$

$V_L = IX_L = (25mA)(2\pi fL)\angle90 = (25mA)(2\pi\times25\times10^{3}\times10\times10^{-3})\angle{+90}$ (ωL)

$= 39.25\angle90$

$V_C = IX_C = (25mA)\left(\frac{1}{2\pi fC}\right)\angle{-90} = (25mA)\left(\frac{1}{2\pi\times25\times10^{3}\times4.05\times10^{-9}}\right)\angle{-90}$

$= 39.31\angle{-90}$

5. Refer to the circuit of Figure 2.
   a. Determine the resonant frequency expressed as ω (rad/s) and f(Hz).
   b. Calculate the total impedance, Z_T, at resonance.
   c. Solve for current I at resonance.

Good Luck

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d. Solve for V_R, V_L, and V_C at resonance.

e. Calculate the power dissipated by the circuit and evaluate the reactive powers, Q_C and Q_L.

f. Find the quality factor, Q_S, of the circuit.

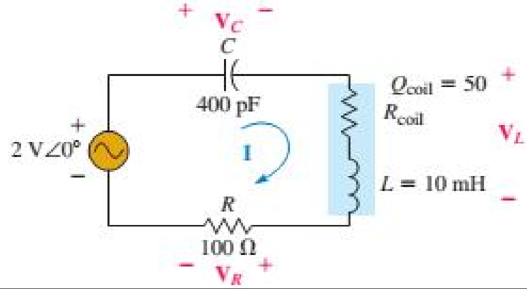

Figure (2)

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\times10^{-12})(10\times10^{-3})}} = 500\,k\,rad/s$

$f = \frac{\omega}{2\pi} = 79.617\,kHz$

b- $Z_t|_{res} = R + R_{coil} = 100 + R_{cl}$

$\because Q = \frac{\omega L}{R_{coil}} \therefore R_{cl} = \frac{\omega L}{Q} = \frac{500\times10^{3}\times10\times10^{-3}}{50} = 100$

$\therefore Z_t = R = 100 + 100 = 200\angle 0$

c- $I_{res} = \frac{V}{Z_{tres}} = \frac{V}{R_t} = \frac{2}{200} = 0.01\,A\angle 0 = 10\,mA\angle 0$

d- $V_R = IR\angle 0 = (0.01)(100) = 1\angle 0$

$V_C = IX_C = (0.01)(\frac{1}{\omega C})\angle -90 = 0.01 \frac{1}{(500)(10^3)(400)(10^{-12})}\angle -90 = 50\angle -90$

$V_L = I(X_L + R) = (0.01)(R + jX_L)$

$= (0.01)(100 + j(500\times10^{3})(10\times10^{-3})) = (0.01)(100 + j5000)$

$= 1 + j50 = \sqrt{1^2 + 50^2}\angle \tan^{-1}(\frac{50}{1}) = 50.01\angle 88.85^\circ$

e- $P = VI = (2)(0.01) = 0.02 = 20\,mW$

$Q_C = I^2 X_C = (0.01)(50) = 0.5\ VAR$

$Q_L = I^2 X_L = (0.01)(50.01) = 0.501\ VAR$

$Q = \frac{\omega L}{R_t} = \frac{(500\times10^{3})(10\times10^{-3})}{200} = 25$

6. Refer to the circuit of Figure 3.

   a. Find ω_S, Q, and BW (in radians per second).

   b. Calculate the maximum power dissipated by the circuit.

   c. From the results obtained in (a) solve for the approximate half-power frequencies, $\omega 1$ and $\omega 2$.

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d. Calculate the actual half-power frequencies, ω1 and ω2, using the component values and the appropriate equations.

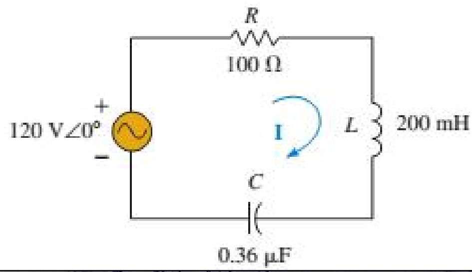
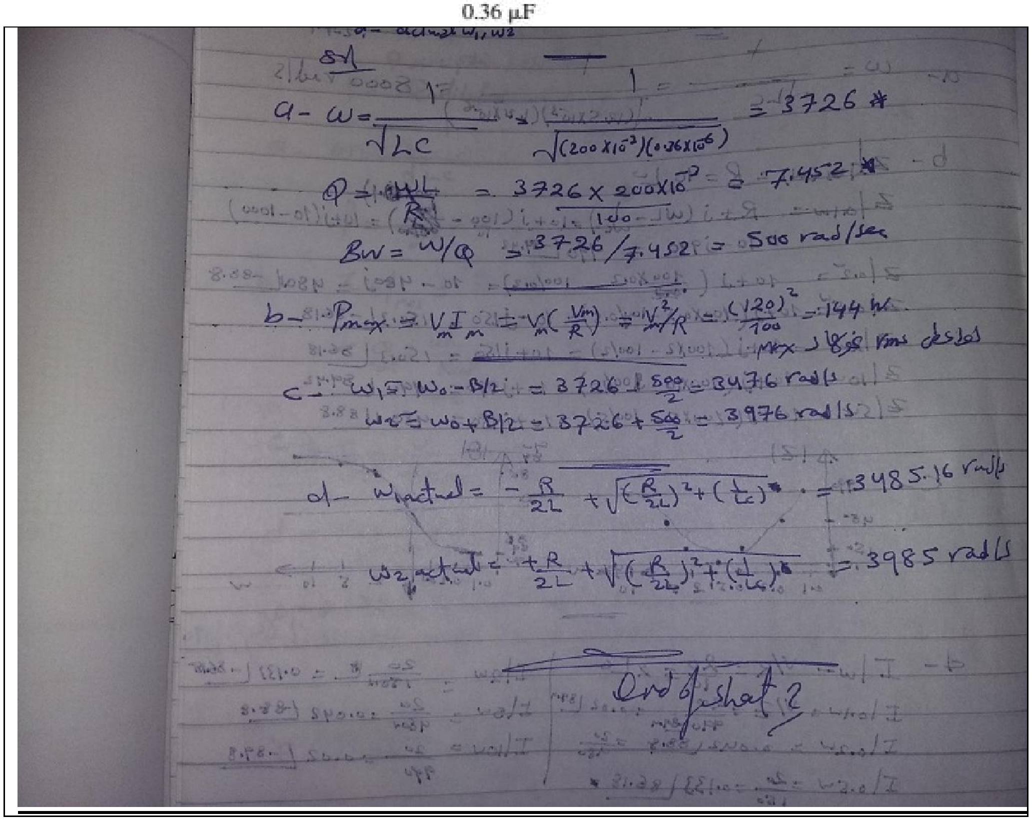


Figure (3)



Sol

a- $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(200\times10^{-3})(0.36\times10^{-6})}} = 3726$

$Q = \dfrac{\omega L}{R} = \dfrac{3726 \times 200\times10^{-3}}{100} = 7.452$

$BW = \omega/Q = 3726/7.452 = 500$ rad/sec

b- $P_{max} = V_m I_m = V_m\left(\dfrac{V_m}{R}\right) = V_m^2/R = \dfrac{(120)^2}{100} = 144$ W

c- $\omega_1 = \omega_0 - B/2 = 3726 - \dfrac{500}{2} = 3476$ rad/s

$\omega_2 = \omega_0 + B/2 = 3726 + \dfrac{500}{2} = 3976$ rad/s

d- $\omega_{1\,actual} = -\dfrac{R}{2L} + \sqrt{\left(\dfrac{R}{2L}\right)^2 + \left(\dfrac{1}{LC}\right)} = 3485.16$ rad/s

$\omega_{2\,actual} = +\dfrac{R}{2L} + \sqrt{\left(\dfrac{R}{2L}\right)^2 + \left(\dfrac{1}{LC}\right)} = 3985$ rad/s

End of sheet 2

Good Luck